L-Resonator Bandstop Filters

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***Abstract*—A design procedure is presented for narrow bandstop filters using TEM transmission line L resonators, which are intermediate of the gap- and parallel-coupled resonators typically used. In this configuration, parallel coupling occurs over a portion of the resonator length, with the remaining resonator length forming a stub. The grounded end of the resonator may be in either the coupled or stub portion of the resonator.**

I. Introduction

IN THE two most common configurations for TEM narrow bandstop filters (Fig. 1), a transmission line is coupled to grounded bandstop resonators by either capacitive gaps [1] or by parallel-line couplings [2], [3]; the structures as shown consist of stripline resonators between parallel ground planes. Capacitively coupled resonators are modeled with a lumped capacitor in series with a transmission-line inductor (shorted transmission line) which is less than a quarter-wavelength ($\lambda/4$) long at resonance; approximate methods must be used for synthesis. Parallel-coupled resonators are modeled with commensurate (equal-length transmission-line) elements, each of which is $\lambda/4$ long at resonance. The model includes a unit element (two-port transmission line) in cascade with either a series commensurate inductor or a shunt commensurate capacitor (open-circuited line); approximate methods or exact synthesis with Richards' frequency transformation ($S = j\tan\theta$ where θ is the commensurate electrical length) may be used. Both filter configurations are typically used to realize conventional, e.g., Chebyshev, responses with all resonant frequencies the same and nominal (at resonance) spacings between resonators of $\lambda/4$. The capacitively-coupled configuration can also be used to realize a general bandstop filter response having different resonant frequencies and spacings [4], including an elliptic function response.

Fig. 2 shows a five-resonator filter in another configuration which is also suitable for narrow bandstop applications, and is the subject of this present discussion. These resonators are intermediate of the two approaches discussed above: each resonator has a parallel-line coupling with an electrical length of less than $\lambda/4$ at resonance, a shunt stub, and an uncoupled portion of the through line to provide additional electrical length between resonators as required. The grounded end may be in either the coupled portion or, as shown in Fig. 2, the stub portion of the resonator; because of the shape, either type is called an "L resonator" here. Coupled and stub line lengths can be arbitrarily chosen with some limitations, and can vary between resonators. Using the L-resonator configuration, the selection of design parameters is greatly increased. The filter designer can incorporate other criteria such as imposed dimensional limitations, power handling, tuning access, etc., more readily into the final design with the increased flexibility and choices allowed by L resonators. As with filters having capacitively coupled resonators, those with L resonators can be designed to meet a general bandstop filter response. Except for particular responses and configurations which permit a commensurate-line synthesis, approximate methods must be used.

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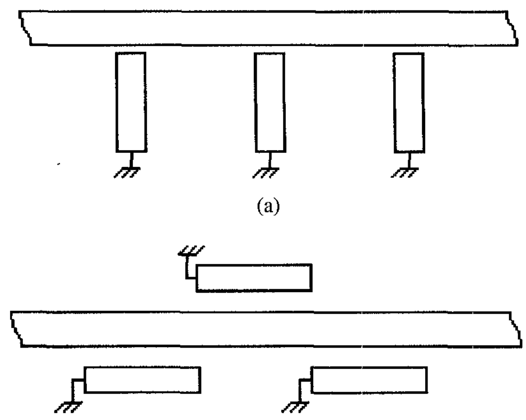

Fig. 1. Bandstop filters with (a) capacitive couplings and (b) parallel couplings.

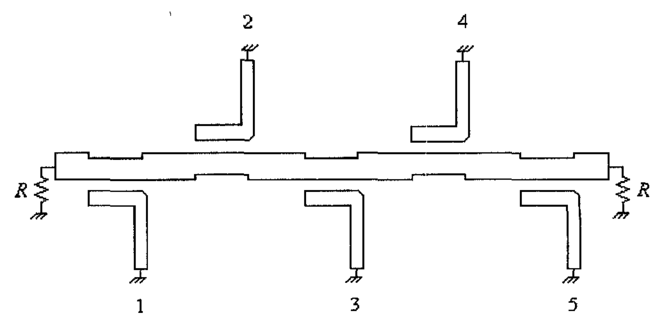

Fig. 2. Bandstop filter with shunt-connected L resonators.

II. Coupled-Line Equivalent Circuits

Saito's equivalent circuits (see [5, Table VII.3]) for the TEM coupled line network consisting of a through line with input and output ports, a coupled line with an arbitrary termination at one end, and either a short or open at the other end are shown in Figs. 3 and 4, respectively; also shown are synthesis and analysis equations. In Fig. 3, the coupled line with a short results in a unit element of characteristic admittance Y_0 and an impedance-transformed series connection of the arbitrary

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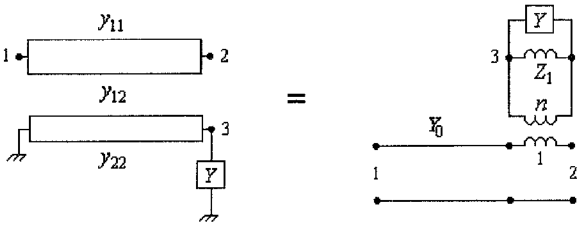
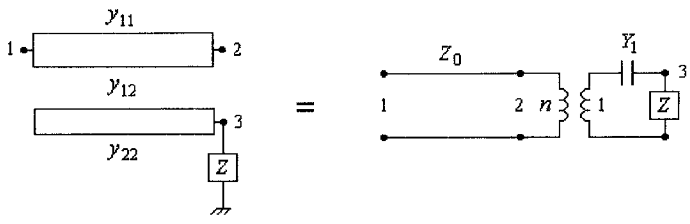


$$y_{11} = Y_0 \qquad Y_0 = y_{11}$$

$$y_{12} = \frac{Y_0}{n} \qquad n = \frac{y_{11}}{y_{12}}$$

$$y_{22} = \frac{1}{Z_1} + \frac{Y_0}{n^2} \qquad Z_1 = \frac{y_{11}}{y_{11}y_{22} - y_{12}^2}$$

Fig. 3. TEM and equivalent circuits for series connection.



$$y_{11} = \frac{1}{Z_0} + \frac{Y_1}{n^2} \qquad Z_0 = \frac{y_{22}}{y_{11}y_{22} - y_{12}^2}$$

$$y_{12} = \frac{Y_1}{n} \qquad n = \frac{y_{22}}{y_{12}}$$

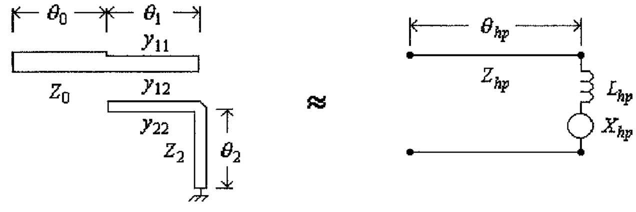
$$y_{22} = Y_1 \qquad Y_1 = y_{22}$$

Fig. 4. TEM and equivalent circuits for shunt connection.

termination Y in parallel with a commensurate inductor of characteristic impedance Z_1. In Fig. 4, the coupled line with an open results in a unit element of characteristic impedance Z_0, and an impedance-transformed shunt connection of the arbitrary termination Z in series with a commensurate capacitor of characteristic admittance Y_1.

A series or shunt bandstop L resonator will be created by replacing the arbitrary termination with a stub which represents a transmission-line capacitor or inductor, respectively, having an electrical length of less than $\lambda/4$ at the respective shunt or series resonant frequency. Note however that the stub length does not have to be commensurate with the coupled line pair; in one extreme (electrical length approaches zero) it would be a lumped element.

In the equivalent circuits, the transformer turns ratios are $n = y_{11}/y_{12}$ (series connection, Fig. 3) and $n = y_{22}/y_{12}$ (shunt connection, Fig. 4). For a physically realizable TEM coupled-line network, $n \geq 1$; for very light coupling, $n^2 \gg 1$, which permits the realization of narrow bandstop resonators using stubs having practical values of characteristic impedance. If the terminating immittances Y or Z in Figs. 3 and 4, respectively, are identically zero, then the equivalent circuits represent the parallel-coupled commensurate-line resonators used in the filter of Fig. 1(b).



$$\theta_0 = \theta_{hp} - \theta_1 \qquad Z_0 = Z_{hp}R \qquad \frac{y_{12}}{n} = \frac{w_p}{L_{hp}R}\,\frac{\dfrac{\theta_1}{\sin 2\theta_1} + \dfrac{\theta_2}{\sin 2\theta_2}}{\tan\theta_1}$$

$$y_{11} = \frac{1}{Z_0} + \frac{y_{12}}{n} \qquad Z_2 y_{22} = \cot\theta_1 \cot\theta_2 \qquad y_{12} = \sqrt{y_{22}}\sqrt{\frac{y_{12}}{n}}$$

Fig. 5. Bandstop L-resonator section and corresponding normalized high-pass prototype section.

In Figs. 3 and 4, the y elements are the characteristic admittances of the coupled line pairs, defined by $y_{ij} = \nu C_{ij}$ where the C_{ij} are the distributed static capacitances of the coupled line network and ν is the speed of light in the medium of propagation [6]. Depending on the method used to design the specific resonators, the y elements may be easily converted to even and odd mode impedances (see [7, Table 5.05-1]) or to the dimensionless ratios C_{ij}/ε, where ε is the dielectric constant in the medium of propagation [2].

III. Shunt-Connected L-Resonator Design

A. General Design

A single section of a shunt-connected L-resonator bandstop filter is shown in Fig. 5, with the corresponding highpass prototype section obtained from narrow bandstop synthesis [4]. The normalized prototype section is characterized by a frequency independent transmission line of characteristic impedance Z_{hp} and constant electrical length θ_{hp} an inductance L_{hp} and a frequency-independent reactance X_{hp} which resonates L_{hp} at the correct point in the stopband. (Frequency independent elements are realized in the synthesis of prototype networks using the narrow-band approximation of circuit theory [8].) The highpass prototype is normalized both in impedance, with unit terminating resistances, and in frequency, with the normalized highpass frequency variable ω', placing the prototype response passband corners at $\omega' = \pm 1$ and the stopband within the range $-1 < \omega' < 1$.

Also shown in Fig. 5 are approximate design equations for calculating the L-resonator parameters from those of the prototype; these equations were obtained by equating normalized reactances and reactance slopes at resonance. To use the design equations, the following must also be specified: the terminating resistance R (e.g., 50 Ω), the nominal electrical line lengths θ_1 and θ_2 and the fractional bandwidth $w_p = bw/f_p$, where bw is the equiripple bandwidth and f_p is the loss pole (series-resonant) frequency of the resonator. In applying the equations, one may specify either the characteristic impedance of the transmission line inductor, Z_2 (e.g., 75 Ω) or the characteristic admittance y_{22}, and use $Z_2 y_{22} = \cot\theta_1 \cdot \cot\theta_2$ (the resonance equation) to calculate the unspecified value; if both y_{22} and Z_2

TABLE I
FIVE-RESONATOR ELLIPTIC-FUNCTION BANDSTOP FILTER

Center Frequency: 2 Ghz — Passband: 1.1 SWR max, 33.78 Mhz wide — Stopband: 40 dB min, 20 Mhz wide

Terminations: $R = 50\ \Omega$

		Uncoupled Through Lines		Coupled-line Pair			Shunt Stub	
Resonator	f_p, Mhz	θ_0, deg*	Z_0, Ω	θ_1, deg*	Z_{0e}, Ω	Z_{0o}, Ω	θ_2, deg*	Z_2, Ω
1	1990.398	Arbitrary**	50	30.15	57.73	42.27	59.68	50
2	1993.712	40.66	50	30.10	61.89	38.11	58.72	50
3	2000.000	53.67	50	30.00	62.58	37.42	58.35	50
4	2006.288	66.42	50	29.91	61.89	38.11	58.35	50
5	2009.602	79.39	50	29.86	57.73	42.27	59.11	50

*All electrical lengths are at center frequency

**This length of line may be set to a convenient value for design.

are specified, a simple iterative procedure discussed in Section IV may be used. As long as $y_{11} > y_{12}$ and $y_{22} > y_{12}$, the circuit is realizable using a coupled-line pair.

B. Special Cases

$\theta_2 = \theta_1$: If the transmission line inductor is made commensurate with the coupled-line pair, two of the equations simplify to

$$\frac{y_{12}}{n} = \frac{w_p}{L_{hp}R}\frac{\theta_1}{\sin^2\theta_1}$$
$$Z_2 y_{22} = \cot^2\theta_1.$$

$\theta_1 + \theta_2 = \pi/2$: If the sum of the line lengths are a quarter-wavelength at resonance, the two equations become

$$\frac{y_{12}}{n} = \frac{w_p}{L_{hp}R}\frac{\frac{\pi}{4}}{\sin^2\theta_1}$$
$$Z_2 y_{22} = 1.$$

$y_{22} = y_{11}$: With this choice, a symmetric coupled-line pair will result. The even and odd mode characteristic impedances Z_{0e} and Z_{0o} are

$$Z_{0e} = \frac{1}{y_{11} - y_{12}}$$
$$Z_{0o} = \frac{1}{y_{11} + y_{12}}.$$

IV. DESIGN EXAMPLE

This example is a five-resonator elliptic-function bandstop filter, in the configuration shown in Fig. 2, with the following requirements: operation in a $50\,\Omega$ system with a center frequency of 2 GHz, 40 dB minimum stopband width of 20 MHz, and 1.1 SWR equiripple passband width of 33.78 Mhz. For each of the L resonators, the coupled line pairs were chosen to be symmetrical, and at the loss pole frequency of each the electrical lengths were chosen to be $\theta_1 = 30°$ for the coupled lines, and, initially, $\theta_2 = 60°$ for the stubs. The parameters $y_{22} = y_{11}$, and y_{12} for each coupled-line pair, and Z_2 for each stub, were calculated and resulted in a set of Z_2 values within the range 46.7 to 48.8 Ω.

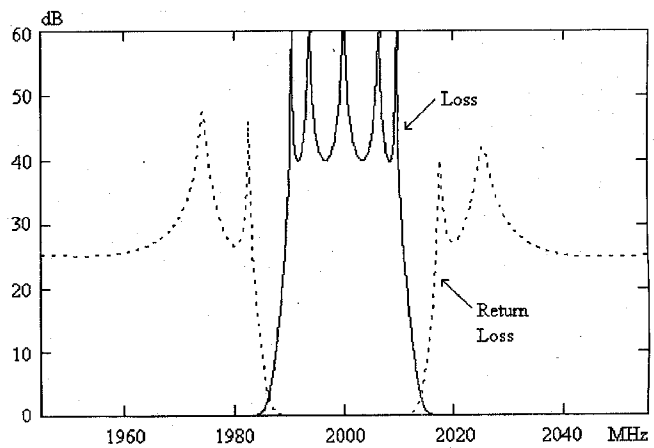


Fig. 6. Theoretical loss and return loss of degree 5 elliptic-function L-resonator bandstop filter.

An iterative procedure was followed in order achieve a set of Z_2 values equal to $50\,\Omega$: setting each Z_2 to 50 Ω, the resonance equation $Z_2 y_{22} = \cot\theta_1 \cdot \cot\theta_2$ was solved for a new set of θ_2 values, which were then used to recalculate the resonator parameters. Repeating this procedure several times, the recalculated designs converged to $Z_2 = 50\ \Omega$ for all stubs. The final design is summarized in Table I. The theoretical response is shown in Fig. 6, and is close to that of the highpass prototype.

V. Conclusion

The *L*-resonator bandstop filter configuration provides increased options in the design of narrow bandstop filters using TEM transmission-line elements. Using the approximate design equations presented for the shunt-connected resonator, bandstop filters realizing the most general types of responses can be designed with a broad range of choices in physical parameters. Similar results may be obtained for series connected *L* resonators. With the appropriate layout, a bandstop filter can be constructed with both shunt- and series-connected resonators. If circumstances require that both ends of the *L* resonator be short or open circuits, the stub portion could be lengthened to between 90 and 180 degrees with appropriate modifications to the design equations.

Acknowledgment

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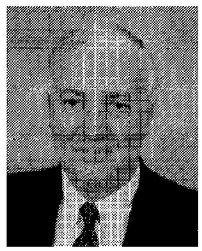

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